## Forces in Action Density

1. The diagram below shows an object submerged in water.


The object is stationary in the water.
Which statement about the upthrust acting on the object is correct?

A It is zero.
B It is equal to the mass of the object.
C It is equal to the weight of the object.
D It is equal to the volume of the water displaced.

Your answer $\square$
2. The flat end of a uniform steel cylinder of weight 7.8 N is glued to a horizontal ceiling. The cylinder hangs vertically. The breaking stress for the glue is 130 kPa


The glue only just holds the cylinder to the ceiling.
What is the cross-sectional area of the cylinder?

A $\quad 6.0 \times 10^{-2} \mathrm{~m}^{2}$
B $\quad 6.0 \times 10^{-5} \mathrm{~m}^{2}$
C $\quad 1.7 \times 10^{-2} \mathrm{~m}^{2}$
D $\quad 1.7 \times 10^{1} \mathrm{~m}^{2}$
$\square$

### 3.2 Forces in Action - Density

3. A student is investigating the resistance of a conducting putty.

The density of conducting putty is $5300 \mathrm{~kg} \mathrm{~m}^{-3}$. The student has a 100 g sample of this putty. Show that the volume $V$ of the sample is about $1.9 \times 10^{-5} \mathrm{~m}^{3}$.
4. A cylinder of wood is placed in water.

The density of the wood is $6.0 \times 10^{2} \mathrm{~kg} \mathrm{~m}^{-3}$. The density of water is $1.0 \times 10^{3} \mathrm{~kg} \mathrm{~m}^{-3}$.
What fraction of the volume of the cylinder is below the water line?

A 0.2
B $\quad 0.4$
C $\quad 0.6$
D $\quad 1.0$

Your answer $\square$
5. Which physical quantity has the same base units as energy?
A. moment
B. momentum
C. force
D. pressure

Your answer $\square$
6. Gas under pressure is forced into a pipe formed into a $U$ shape bend. A liquid of density $\rho$ is pushed up the right hand side so there is a difference $h$ in the height on each side, measured along the side of the tube. The pipe on the right hand side is inclined at an angle $\theta$ to the vertical and it has a cross sectional area double that on the left side.


What is the pressure of the gas in excess of atmospheric pressure?
A. $h \rho g$
B. $2 h \rho g$
C. $h \rho g \cos \theta$
D. $2 h \rho g \cos \theta$

7. What is the best estimate for the density of steel?
A. $\quad 10^{-3} \mathrm{~g} \mathrm{~cm}^{-3}$
B. $10^{3} \mathrm{~g} \mathrm{~cm}^{-3}$
C. $1 \mathrm{~kg} \mathrm{~m}^{-3}$
D. $10^{3} \mathrm{~kg} \mathrm{~m}^{-3}$
8. A group of civil engineers are assessing whether or not to use solid concrete pillars or hollow metal tubes to support a building. One such tube is shown below. The tube is placed on a horizontal surface. The tube is made of metal of thickness $t$. The tube has height $h$ and a mean internal radius $R$. The radius $R \gg$ thickness $t$.


A heavy metal block of mass $m$ is placed on top of the tube.
What is the approximate pressure $p$ acting on the tube?

A $\quad p=\frac{m g}{2 \pi R t}$
B $\quad p=\frac{m g}{\pi R^{2}}$
C $\quad p=\frac{m g}{\pi R^{2} h}$
D $p=\frac{m g}{\pi R^{2} t}$
Your answer $\square$
9. A small block of wood is placed in deep water.

The block is at rest with $80 \%$ of its volume under the surface of water.
The weight of the block is 6.0 N .
What is the upthrust acting on the block?

A 0
B $\quad 1.2 \mathrm{~N}$
C $\quad 4.8 \mathrm{~N}$
D $\quad 6.0 \mathrm{~N}$

### 3.2 Forces in Action - Density

Your answer
10. A brick of mass $m$ has sides of lengths $a, b$ and $c$, where $a<b<c$. The brick is placed on a horizontal table such that the pressure it exerts on the table is a maximum.

What is the maximum pressure $p$ acting on the table?

A $\quad p=\frac{m g}{a b}$
B $\quad p=\frac{m g}{a c}$
C $p=\frac{m g}{b c}$
D $\quad p=\frac{m g}{a b c}$

Your answer

11. A bottle cork floats on water. It is partially submerged in the water.


Which of the following statements is / are true?

1. The net force acting on the cork is equal to the weight of the water displaced.
2. The weight of the cork is equal to the upthrust on the cork.
3. The upthrust on the cork is equal to the mass of the water displaced.
A. 1, 2 and 3
B. Only 2 and 3
C. Only 3
D. Only 2

Your answer $\square$
12. The table below shows the measurements recorded by a student for a solid metal sphere. The absolute uncertainties in the mass of the sphere and in its radius are also shown.

| mass | $100 \pm 6 \mathrm{~g}$ |
| :--- | :--- |
| radius | $1.60 \pm 0.08 \mathrm{~cm}$ |

What is the percentage uncertainty in the density of the sphere?
A. $1 \%$
B. $11 \%$
C. $16 \%$
D. $21 \%$

Your answer $\square$
13. This question is about helium in the atmosphere of the Earth.

Experiment shows that most of the Earth's atmosphere is contained within a very thin shell around the surface of the Earth. Less than $0.0001 \%$ of this is helium.

Assume that the Earth's atmosphere has a constant density $\rho$ of $1.3 \mathrm{~kg} \mathrm{~m}^{-3}$. The atmospheric pressure at sea level is $1.0 \times 10^{5} \mathrm{~Pa}$.
Show that the depth of the atmosphere under these conditions would be about 8 km .
14. A cylindrical cup of internal diameter 7.0 cm and height 8.5 cm is filled to the top with water.


### 3.2 Forces in Action - Density

The density of water is $1000 \mathrm{~kg} \mathrm{~m}^{-3}$. The mass of one mole of water is 18 g . The specific heat capacity of water is $4200 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$.

Show that the mass of water in the cup is approximately 0.3 kg .
15. A swimming pool designer investigates the depth $d$ below a water surface reached by a diver when diving from
a height $h$ above the water surface.
The designer models the diver as a uniform wooden cylinder.
The experimental arrangement is shown in Fig. 18.1.


Fig. 18.1
The graph of Fig. 18.3 shows the depth $d$ reached for different initial drop height $h$.


Fig. 18.3
The designer is required to double the height of a diving board for an existing swimming pool. He suggests that the depth of the pool also needs to be doubled.
Use Fig. 18.3 to explain whether you agree with this suggestion.

16 (a). A group of students are conducting an experiment in the laboratory to determine the value of absolute zero by heating a fixed mass of gas. The volume of the gas is kept constant. Fig. 17.1 shows the arrangement used by the students.


Fig. 17.1
The gas is heated using a water bath. The temperature $\theta$ of the water is increased from $5^{\circ} \mathrm{C}$ to $70^{\circ} \mathrm{C}$. The temperature of the water bath is assumed to be the same as the temperature of the gas. The pressure $p$ of the gas is measured using a pressure gauge.

The results from the students are shown in a table.

| $\theta /{ }^{\circ} \mathbf{C}$ | $\boldsymbol{p} / \mathbf{k P a}$ |
| :--- | :--- |
| $5 \pm 1$ | $224 \pm 3$ |
| $13 \pm 1$ | $231 \pm 3$ |
| $22 \pm 1$ | $238 \pm 3$ |
| $35 \pm 1$ | $248 \pm 3$ |
| $44 \pm 1$ |  |
| $53 \pm 1$ | $262 \pm 3$ |
| $62 \pm 1$ | $269 \pm 3$ |
| $70 \pm 1$ | $276 \pm 3$ |

Describe and explain how the students may have made accurate measurements of the temperature $\theta$.
(b). Fig. 17.2 shows the pressure gauge. Measurements of $p$ can be made using the kPa scale or the psi (pounds per square inch) scale. The students used the psi scale to measure pressure and then converted the reading to pressure in kPa .


Fig. 17.2
i. Suggest why it was sensible to use the psi scale to measure $p$.

ii. The students made a reading of $p$ of $37.0 \pm 0.5$ psi when $\theta$ was $44 \pm 1^{\circ} \mathrm{C}$.

Convert this value of $p$ from psi to kPa . Complete the table for the missing value of $p$. Include the absolute uncertainty in $p$.

1 pound of force $=4.448 \mathrm{~N}$
1 inch $=0.0254 \mathrm{~m}$
(c). Fig. 17.3 shows the graph of $p$ against $\theta$.


Fig. 17.3
i. Plot the missing data point and the error bars on Fig. 17.3.
ii.

Explain what is meant by absolute zero. Describe how Fig. 17.3 can be used to determine the value of absolute zero.
Determine the value of absolute zero. You may assume that the gas behaves as an ideal gas.
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3.2 Forces in Action - Density

(d). Describe, without doing any calculations, how you could use Fig. 17.3 to determine the actual uncertainty in the value of absolute zero in (c)(ii).
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(e). The experiment is repeated as the water bath quickly cools from $70^{\circ} \mathrm{C}$ to $5^{\circ} \mathrm{C}$. Absolute zero was found to be $-390^{\circ} \mathrm{C}$.

Compare this value with your value from (c)(ii) and explain why the values may differ. Describe an experimental approach that could be taken to avoid systematic error in the determination of absolute zero.
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17. A student is designing a three-legged wooden stool as shown in Fig. 2.2.


Fig. 2.2

The stool must be able to support the weight of an adult.
The maximum compressive stress of the wood is 2.3 MPa .
Estimate the minimum cross-sectional area $A$ of one leg.

$$
A=.
$$

$\mathrm{m}^{2}$ [3]

18 (a). A toy rocket is made from a 1.5 litre plastic bottle with fins attached for stability.
The bottle initially contains 0.30 litres of water, leaving 1.2 litres of trapped air at a temperature of $17^{\circ} \mathrm{C}$. A pump is used to increase the pressure of the air within the plastic bottle to $2.4 \times 10^{5} \mathrm{~Pa}$ at the start of lift-off.

Fig. 1.1 shows the rocket at the start of lift-off.
1 litre $=10^{-3} \mathrm{~m}^{3}$


Fig. 2.1

The trapped air pushes the water downwards out of the hole, causing the rocket to rise. The temperature of this air remains constant.

### 3.2 Forces in Action - Density

Calculate the final pressure of the trapped air just before all the water has been released.

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final pressure =
(b). Here is some data on the toy rocket.
mass of empty bottle and fins \(=0.050 \mathrm{~kg}\) area of cross-section of hole \(=1.1 \times 10^{-4} \mathrm{~m}^{2}\) initial pressure of trapped air \(=2.4 \times 10^{5} \mathrm{~Pa}\) atmospheric pressure \(=1.0 \times 10^{5} \mathrm{~Pa}\) density of water \(=1.0 \times 10^{3} \mathrm{~kg} \mathrm{~m}^{-3}\)
i. Use the data above to show that the upwards force on the rocket at the start of lift-off is about 15 N
ii. Hence calculate the initial vertical acceleration of the rocket.

19 (a). A meteorological balloon rises through the atmosphere until it expands to a volume of \(1.0 \times 10^{6} \mathrm{~m}^{3}\), where the pressure is \(1.0 \times 10^{3} \mathrm{~Pa}\). The temperature also falls from \(17^{\circ} \mathrm{C}\) to \(-43^{\circ} \mathrm{C}\).

The pressure of the atmosphere at the Earth's surface \(=1.0 \times 10^{5} \mathrm{~Pa}\).
Show that the volume of the balloon at take off is about \(1.3 \times 10^{4} \mathrm{~m}^{3}\).
(b). The balloon is filled with helium gas of molar mass \(4.0 \times 10^{-3} \mathrm{~kg} \mathrm{~mol}^{-1}\) at \(17^{\circ} \mathrm{C}\) at a pressure of \(1.0 \times 10^{5}\) Pa .

\section*{Calculate}
i. the number of moles of gas in the balloon
number of moles \(=\) \(\qquad\)
ii. the mass of gas in the balloon.

> mass =
\(\qquad\) kg [1]
(c). The internal energy of the helium gas is equal to the random kinetic energy of all of its molecules.

When the balloon is filled at ground level at a temperature of \(17^{\circ} \mathrm{C}\), the internal energy is 1900 MJ .
Estimate the internal energy of the helium when the balloon has risen to a height where the temperature is \(-43^{\circ} \mathrm{C}\).
internal energy =
(d). The acceleration of the balloon and its instruments at the Earth's surface as it is released is \(27 \mathrm{~m} \mathrm{~s}^{-2}\).

The density of the air at the Earth's surface is \(1.3 \mathrm{~kg} \mathrm{~m}^{-3}\).
Calculate the total mass \(M\) of the helium-filled balloon and its load.
20. Determine the average density of the Earth. The radius of the Earth is 6400 km .
average density \(=\) \(\qquad\) \(\mathrm{kg} \mathrm{m}^{-3}\)
21. Fig. 6.1 shows a uniform metal cylinder of weight 7.0 N . The cylinder has length 100 mm and diameter 32 mm.


Fig. 6.1 (not to scale)
Calculate the density \(\rho\) of the metal.
\(\rho=\)
\(\mathrm{kg} \mathrm{m}^{-3}[3]\)
22. Civil engineers are designing a floating platform to be used at sea. Fig. 4.1 shows a model for one section of this platform, a sealed metal tube of uniform cross-sectional area, loaded with small pieces of lead, floating upright in equilibrium in water.


Fig. 4.1

The tube has length 300 mm and diameter 50 mm . The total mass of the lead and tube is 0.50 kg . Show that the length / of tube above the surface is more than 40 mm .
density of water \(=1000 \mathrm{~kg} \mathrm{~m}^{-3}\)
23. This question is about upthrust and other forces acting on a sealed hollow tube in water.

One end of a string is attached to the bottom of the tube and the other end of the string is attached to the bottom of the container. The string exerts a downward force \(F\) on the tube. At time \(t=0\), the tube is half submerged in the water, as shown in Fig. 23.1.


Fig. 23.1

The container is slowly filled with water at a constant rate until the container is full. Fig. 23.2 shows the graph of \(F\) against time \(t\).


Fig. 23.2

By considering the forces acting on the tube, explain the general shape of the graph shown in Fig. 23.2.
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24 (a). Fig. 4.1 shows a uniform wooden cylinder.


Fig. 4.1

The cylinder has height 12.0 cm and diameter 2.9 cm .
The density of the wood is \(400 \mathrm{kgm}^{-3}\).
i. Show that the cross-sectional area of the wooden cylinder is about \(6.6 \times 10^{-4} \mathrm{~m}^{2}\).
ii. Calculate the weight \(W\) of the wooden cylinder.
\(\qquad\)
(b). A student places the wooden cylinder in a beaker of water so that it floats. The vertical distance between the water surface and the bottom of the cylinder is \(y\), as shown in Fig. 4.2.


Fig. 4.2 (not to scale)

\subsection*{3.2 Forces in Action - Density}

The density of water is \(1000 \mathrm{~kg} \mathrm{~m}^{-3}\).
Calculate the distance \(y\).
\[
y=
\]
(c). The student repeats the experiment, but replaces the water with oil of density \(900 \mathrm{~kg} \mathrm{~m}^{-3}\). The cylinder will still float.

Calculate the new distance \(y\). Explain your answer.
Calculation:
\[
y=
\]

\section*{Explanation:}
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25. A long wooden cylinder is placed into a liquid and it floats as shown.


The length of the cylinder below the liquid level is 15 cm .
i. State Archimedes' principle .
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\(\qquad\)
ii. The pressure exerted by the liquid alone on the bottom of the cylinder is \(1.9 \times 10^{3} \mathrm{~Pa}\).

Calculate the density \(\rho\) of the liquid.
\(\rho=\) \(\qquad\) \(\mathrm{kg} \mathrm{m}^{-3}\) [2]

\subsection*{3.2 Forces in Action - Density}

26 (a). A student measures the diameter of a ball in different directions.
The student's results are:
\[
2.43 \mathrm{~cm} \quad 2.54 \mathrm{~cm} \quad 2.59 \mathrm{~cm}
\]
i. State the name of a suitable measuring instrument to measure the diameter of the ball.
ii. Calculate the mean diameter \(d\) of the ball. Include the absolute uncertainty in \(d\).
\(d=\) \(\qquad\) \(\pm\). \(\qquad\) cm [2]
iii. Show that the volume of the ball is about \(8.4 \times 10^{-6} \mathrm{~m}^{3}\).
iv. The mass of the ball is \(23 \pm 1 \mathrm{~g}\).

Determine the density \(\rho\) of the ball.
Give your answer to an appropriate number of significant figures.
\(\qquad\)
\(\rho=\) \(\mathrm{kg} \mathrm{m}^{-3}\) [2]
v. Determine the percentage uncertainty in \(\rho\).
(b). The 23 g mass ball from (a) is used in an experiment with a spring.

The student measures the unstretched length Lo of a spring as shown in Fig. 3.1.


Fig. 3.1


Fig. 3.2

The student then attaches the ball to the spring and measures the length \(L\) of the spring as shown in Fig. 3.2.

The student's results are:
\(L_{0}=0.078 \mathrm{~m}\) and \(L=0.096 \mathrm{~m}\)
Calculate the force constant \(k\) of the spring.
\(k=\)
\(\mathrm{N} \mathrm{m}^{-1}[3]\)
(c). The 23 g mass ball from (a) and the spring from (b) are now used in an experiment to investigate upthrust.

The ball attached to the spring is lowered into a beaker containing a liquid so that it is totally submerged. The student measures the new length \(L_{N}\) of the spring, as shown in Fig. 3.3.


Fig. 3.3

The length \(L_{N}\) of the spring is now 0.088 m .
i. Calculate the upthrust on the submerged ball.
upthrust \(=\)
N [2]
ii. Calculate the density of the liquid.
\(\mathrm{kg} \mathrm{m}^{-3}\) [2]

\subsection*{3.2 Forces in Action - Density}
27. A swimming pool designer investigates the depth \(d\) below a water surface reached by a diver when diving from
a height \(h\) above the water surface.
The designer models the diver as a uniform wooden cylinder.
The experimental arrangement is shown in Fig. 18.1.


Fig. 18.1
The wooden cylinder has mass \(5.0 \times 10^{-3} \mathrm{~kg}\), diameter \(1.0 \times 10^{-2} \mathrm{~m}\) and length \(7.0 \times 10^{-2} \mathrm{~m}\).
i. Calculate the density of the wood.

> density =
\(\mathrm{kg} \mathrm{m}^{-3}[2]\)
i. Suggest why wood is an appropriate material to model the depth reached by a diver.
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\subsection*{3.2 Forces in Action - Density}

28 (a). Fig. 23.1 shows a metal cylinder of diameter of about 5 cm placed on a horizontal table.


Fig. 23.1
i. State Archimedes' principle.
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\(\qquad\)
ii. Fig. 23.2 shows the metal cylinder hung from a newtonmeter.


Fig. 23.2

The reading on the newtonmeter is 9.0 N .
The cylinder is slowly lowered into water in a beaker until it is completely submerged.
The cylinder does not touch the side or the bottom of the beaker. The newtonmeter reading now is 7.8 N . The density of water is \(1000 \mathrm{~kg} \mathrm{~m}^{-3}\).
Calculate the density \(\rho\) of the metal of the cylinder.
\[
\rho=
\]
(b). Fig. 23.1 shows a metal cylinder of diameter of about 5 cm placed on a horizontal table.


Fig. 23.1

Describe how you can use instruments available in a physics laboratory to determine the pressure exerted by the cylinder on the table. State how you would make your results as precise as possible.
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29. This question is about the operation of an electrically powered shower designed by an electrical firm.

shower head

Fig.1.1

Water moves at constant speed through a pipe of cross-sectional area \(7.5 \times 10^{-5} \mathrm{~m}^{2}\) to a shower head shown in Fig. 1.1. The maximum mass of water which flows per second is \(0.070 \mathrm{~kg} \mathrm{~s}^{-1}\).
i. Show that the maximum speed of the water in the pipe is about \(0.9 \mathrm{~m} \mathrm{~s}^{-1}\). density of water \(=1000 \mathrm{~kg} \mathrm{~m}^{-3}\)
ii. The total cross-sectional area of the holes in the shower head is one quarter that of the pipe. Calculate the maximum speed of the water as it leaves the shower head.

> maximum speed =
\(\qquad\) \(\mathrm{m} \mathrm{s}^{-1}\)
iii. Calculate the magnitude of the force caused by the accelerating water on the shower head.
\[
\text { force }=
\]
iv. Draw on to Fig. 1.1 the direction of the force in (iii).

30 (a). A solid wooden sphere of density \(650 \mathrm{~kg} \mathrm{~m}^{-3}\) has a diameter of 2.8 cm .
i. Describe and explain how the student can measure precisely the diameter of the sphere.
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\(\qquad\)
\(\qquad\)
ii. Show that the mass of the sphere is 0.0075 kg .
iii. The sphere is pushed below the surface of water as shown in Fig. 3.

\subsection*{3.2 Forces in Action - Density}


Fig. 3
Determine the force \(F\) that needs to be applied to the sphere to keep the wooden sphere stationary in this position.
density of water \(=1000 \mathrm{~kg} \mathrm{~m}^{-3}\)
\(\qquad\)
\[
F=
\]

N [2]
(b). A student wishes to investigate how the terminal velocity \(v\) of a metal sphere varies with the radius \(r\) of the sphere as it travels through a liquid.
It is suggested that
\[
v=K r^{2}
\]
where \(K\) is a constant.
Describe with the aid of a suitable diagram how an experiment can be safely conducted, and how the data can be analysed to determine \(K\).
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\subsection*{3.2 Forces in Action - Density}
31. Fig. 18.2 shows a cylinder fully submerged under the water surface before it has come to rest. The cylinder is moving vertically down.


Fig. 18.2
i. Add arrows to Fig. 18.2 to show the three forces acting on the wooden cylinder. Label the arrows.
ii. Describe and explain how the resultant force on the wooden cylinder varies from the moment the cylinder is fully submerged until it reaches its deepest point.
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